$K$ = universal constant in Equation (3)
$M$ = molecular weight
$P$ = pressure, lb./sq. in. abs.
$P_c$ = critical pressure, lb./sq. in. abs.
$P_{c(x)}$ = critical pressure of reference substance $x$, lb./sq. in. abs.
$P_R$ = reduced pressure, dimensionless
$T_c$ = critical temperature, °K.
$T_R$ = reduced temperature, dimensionless
$t$ = temperature, °C.

**Greek Letters**

$\beta_{t,p}$ = viscometer constant at temperature and pressure, sq. cm./ sec.$^2$
$\beta_o$ = viscometer constant corrected to 0°C. and 0 pressure, sq. cm./sec.$^2$
$\theta$ = fall time interval, sec.
$\mu$ = absolute viscosity, poise
$\rho$ = density of fluid, g./cc.
$\rho_c$ = critical density of fluid, g./cc.
$\sigma$ = density of the falling body, g./cc.

**LITERATURE CITED**


1. Barkelew, C. H., J. L. Valentine, and C. O. Hurd, *Chem. Eng. Progr.*, **43**, 25 (1947).
2. Bicher, L. B., Jr., and D. L. Katz, *Ind. Eng. Chem.*, **35**, 754 (1943).
3. Comings, E. W., B. J. Mayland, and R. S. Egly, *Univ. Ill. Eng. Exp. Sta. Bull. Ser.*, No. 354 (1944).
4. Deschner, W. W., and G. G. Brown, *Ind. Eng. Chem.*, **32**, 836 (1940).
5. Eilerts, C. K., *et al.*, "Phase Relationships of Gas-Condensate Fluids," Vol. II, pp. 830-2, American Gas Association, New York (1959).
6. Gerf, S. F., and G. I. Galkov, *J. Tech. Phys.* (*U.S.S.R.*), **11**, 801 (1941).
7. Grunberg, L., and A. H. Nissan, *Nature*, **161**, 170 (1948).
8. Hanson, G. H., *Trans. Am. Inst. Chem. Engrs.*, **42**, 959 (1946).
9. Kay, W. B., *Ind. Eng. Chem.*, **32**, 358 (1940).
10. Lipkin, M. A., J. A. Davison, and S. S. Kurtz, Jr., *ibid.*, **34**, 976 (1942).
11. Matthews, C. S., and C. O. Hurd, *Trans. Am. Inst. Chem. Engrs.*, **42**, 55 (1946).
12. Michels, A., and R. O. Gibson, *Proc. Roy. Soc.* (*London*), **134A**, 288 (1931).
13. Pavlovich, N. V., and D. L. Timrot, *Teploenergetika*, **5**, No. 8, 61 (1958).
14. Rossini, F. D., "Am. Petrol. Inst. Research Proj. 44," Carnegie Press, Pittsburgh, Pa. (1958).
15. Rudenko, N. S., *J. Exp. Theoret. Phys.* (*U.S.S.R.*), **9**, 1078 (1939).
16. Sage, B. H., and W. N. Lacey, *Ind. Eng. Chem.*, **30**, 829 (1938).
17. Sage, B. H., W. D. Yale, and W. N. Lacey, *ibid.*, **31**, 223 (1939).
18. Smith, A. S., and G. G. Brown, *ibid.*, **35**, 705 (1943).
19. Starling, K. E., B. E. Eakin, and R. T. Ellington, *A.I.Ch.E. Journal*, to be published.
20. Swift, G. W., Ph.D. thesis, Univ. Kansas, Lawrence (1959).
21. ———, J. A. Christy, A. A. Heckes, and Fred Kurata, *Chem. Eng. Progr.*, **54**, No. 6, 47 (1958).
22. Swift, G. W., J. A. Christy, and Fred Kurata, *A.I.Ch.E. Journal*, **5**, 98 (1959).
23. Van Wijk, W. R., J. H. Van der Veen, H. C. Brinkman, and W. A. Seeder, *Physica*, **7**, 45 (1940).


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# Suspension of Slurries by Mechanical Mixers

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**The production of essentially homogeneous slurries involves both initial solids suspension and expansion of the particle bed to fill the container. When geometry and solids concentration are held constant, both phenomena are found to be controlled by similar dimensionless groupings of power per unit volume, density, and relative velocity between the fluid and particle. The dimensionless group applicable to bed expansion is shown to be consistent with hydrodynamic theory. Design equations for use with the paddle type of impeller are presented.**

Slurries have recently been used in an increasing number of industrial situations. An application of major interest is the proposed use of an aqueous thoria-urania slurry as a nuclear reactor fuel. Solving the problems associated with the use of this fuel in a large-sized power plant has been the subject of an extensive research and development program (7, 8).

Maintenance of an essentially homogeneous suspension is one of the prime requisites for successful operation of a slurry-fueled nuclear power plant. Therefore as part of the development program the problems of slurry suspension were investigated in several types of equipment. The mixing studies reported in this paper were one aspect of this investigation. The specific objectives of these mixing experiments were to determine for the turbulent region the factors controlling the production of essentially homogeneous suspensions of solid particles by mechanical mixers and to obtain the information necessary for the design of large mixers suitable for use in plant-sized slurry storage tanks.

Tabular material has been deposited as document No. 6268 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for $2.50 for photoprints or $1.75 for 35-mm. microfilm.

## PREVIOUS STUDIES ON SLURRY AGITATION

Hixson and co-workers (*3* to *6*) in an extensive study of the agitation of slurries of soluble salts determined mass transfer coefficients as a function of mixing Reynolds number and system geometry.

White, Summerford, *et al.* (*14, 15*) and Raghavendra and Mukherji (*11*) studied the distribution of sand in unbaffled tanks using paddle agitators. White *et al.* found that with coarsely sized material the fines were suspended but the large particles remained at the tank bottom. Later studies with more closely sized material of various sizes (0.14 to 0.42 mm.) showed that it was not possible to achieve complete suspension of all the sand particles in their system. They concluded that in an unbaffled system the vortex established at high speeds produces centrifugal forces which are sufficiently strong to combat the forces tending to produce a more uniform concentration.

The mixing of dilute suspensions of insoluble solids in highly viscous liquids was investigated by Hirsekorn and Miller (*2*). The particle size, liquid viscosity, and tank diameter were simultaneously varied so that both the mixing Reynolds number and particle settling rate were maintained approximately constant. The minimum power to achieve suspension was defined as the power input when all particles were lifted from the bottom of the tank. The authors' results can be expressed as

$$(P_s/V_t)(d/D) = \text{constant} \quad (1)$$

It was noticed that just as complete suspension was obtained, the portion of the liquid nearest the surface was devoid of solids. The material below the slurry-fluid interface was however essentially uniform in concentration.

Zwietering (*16*) reported the results of an extensive investigation of stirrer speeds and dimensions required to suspend various slurries. He considered only the question of suspension and reported no quantitative information on the degree of homogeneity of the mixture. Zwietering did note that with rapidly settling particles just at their suspension point a layer of clean liquid exists adjacent to the surface. The investigation included studies with paddles, turbines, vaned disks, and propellers. When one uses Hirsekorn and Miller's (*2*) suspension criterion (no particles remaining on tank bottom), the conditions required for suspension could be correlated by

$$s = \frac{Nd^{0.85}}{\nu^{0.1}\delta^{0.2}(g\Delta\rho/\rho_l)^{0.45}B^{0.13}} = F[(D/d)(a/D)] \quad (2)$$

The effect of $D/d$ and $D/a$ were presented graphically for each of the impeller types studied. For paddle type of impellers the dimensionless characterization group $s$ increased with increasing values of $D/d$ and $a/D$.

In studies of the rate of solution of salt crystals in an agitated tank Kneule (*9*) found that applied power inputs beyond those required just to suspend the particles had little effect on the rate of mass transfer. If the notation of Zwietering (*16*) is used to facilitate comparison, the suspension-power correlation presented by Kneule can be written as

$$A = \frac{B^{1/2}V_t[g(\Delta\rho)]^{3/2}\delta^{1/2}}{P_t g_c \rho_l^{1/2}} \quad (3)$$

Values of A were not presented by Kneule. The correlations of Rushton *et al.* (*12*) show that the mixing power is given by

$$P = N_p \rho_m N^3 d^5/g_c \quad (4)$$

Since for a given geometry $N_p$ is a constant in the fully turbulent region, one can rewrite Equation (3) as

$$K\left(\frac{D}{d}\right)A^{-1/3} = \left(\frac{g_c}{g}\right)\frac{Nd^{2/3}(\rho_l/\rho_m)^{1/6}}{\delta^{1/6}[g(\Delta\rho/\rho_m)]^{1/2}B^{1/6}} \quad (5)$$

If one notes that for most of the systems tested $\rho_m$ does not differ greatly from $\rho_l$, the marked similarity of Equations (2) and (5) is evident.

It is pointed out in the work of Hirsekorn and Miller (*2*) that the suspension of particulate solids involves two distinct problems: the initial suspension of the particles and the complete dispersion of the already suspended solids. At the beginning of this investigation the literature data on the former problem were not in complete agreement, and there was essentially no information applicable to the latter problem in the fully turbulent region.

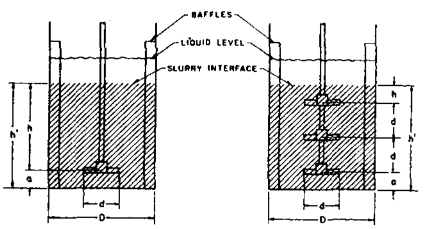


**Fig. 1. Mixing-tank arrangements.**

## EXPERIMENTAL APPARATUS AND MATERIALS

Methods of producing slurry suspensions by rotating mixers were studied in transparent cylindrical tanks having diameters of 5½, 9⅝ and 11⅝ in. Figure 1 shows the arrangement of both single and multiple impeller systems. Each tank was fitted with four baffles extending to the tank bottom and having a baffle width to tank diameter ratio of 1/12. The tanks were mounted so that mixing phenomena might be observed visually through the walls and bottom of the vessel. In all cases studied, six-bladed paddle-type impellers with a width-to-diameter ratio of ⅛ were used. The impellers were 2, 3, and 4 in. in diameter. The mixing vessels were mounted in an adjustable frame so that the impeller height above the vessel bottom might be varied.

A ¼-hp. variable-speed motor was suspended vertically above the vessel by means of a thin wire. Lateral motion was restricted by a loosely fitted radial ball bearing. The power imput to the vessel was computed from measurement of the shaft torque and speed. Torque values were obtained from a calibrated spring scale of appropriate range acting upon a 2-in. lever arm. Speed measurements were obtained from an electronic frequency meter with a photoelectric pickup. An aluminum disk, integral with the motor chuck and painted in alternate dark and light bands, served as an intermittent reflector for the photoelectric pickup. During the course of the experiments the impeller power requirements varied from 0.001 to 0.17 hp., and impeller speeds ranged from 200 to 2,200 rev./min. It is estimated that the maximum error in the computed power was less than 10%.

The mixing studies were conducted with aqueous slurries of thorium oxide and both aqueous and nonaqueous slurries of spherical glass beads. A series of Tyler sieves were used to determine the average diameter of both sizes of glass beads which were studied. All particles of the small-size beads were between 37 and 53 $\mu$ in diameter. The average diameter was therefore taken as 45 $\mu$. The large-sized glass beads were found to have the following size distributions:

| Diameter range, ($\mu$) | Weight, % |
|---|---|
| 88 — 125 | 6.4 |
| 125 — 147 | 9.3 |
| 147 — 177 | 63.2 |
| > 177 | 21.2 |
| | 100.0 |

Microscopic examination of the particles above 177 $\mu$ showed them to have an average diameter of about 185 $\mu$. The average diameter of the large-sized beads was taken as

$$\delta_{avg} = \frac{\Sigma N_i \delta}{\Sigma N_i} \quad (6)$$

By use of the mean diameter in each size range and the relative number of particles in each range $\delta_{avg}$ was computed as 140 $\mu$.

The hindered settling rates of slurries of both the large and small beads were measured as a function of concentration. At any given concentration the ratio of settling rates was found to be in agreement with the ratio of the squares of the average particle diameters. This was taken as a good indication that the computations had given the correct relative values of the average diameters.

The problem of the determination of the average diameter (or free settling rate) of the thoria particles is more complex. Although the thoria is composed of very small particles (all under 4 $\mu$, most under 1 $\mu$), these particles agglomerate to form relatively large flocs. The mixing studies are concerned with the behavior of the flocs rather than the individual particles. Andreasen pipette measurements (without a dispersant) indicated the free

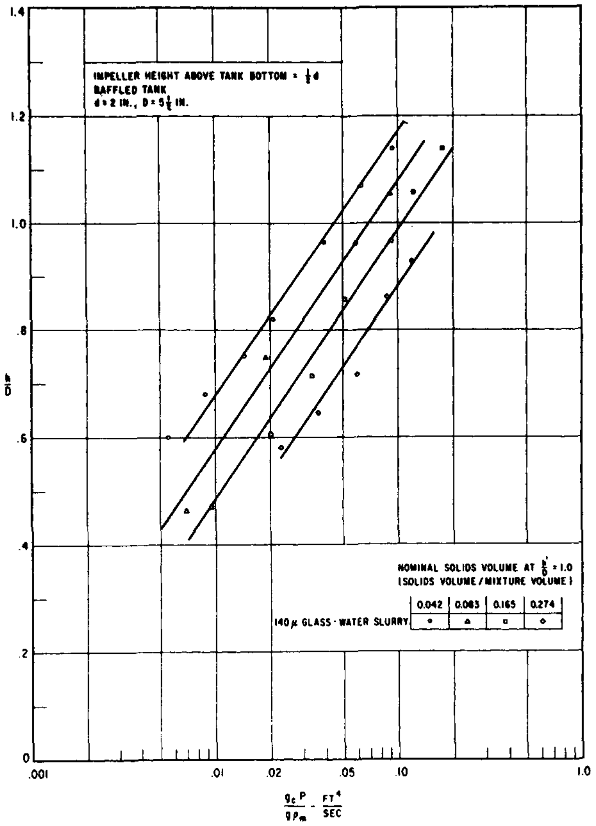


**Fig. 2. Effect of concentration on height of the slurry interface.**

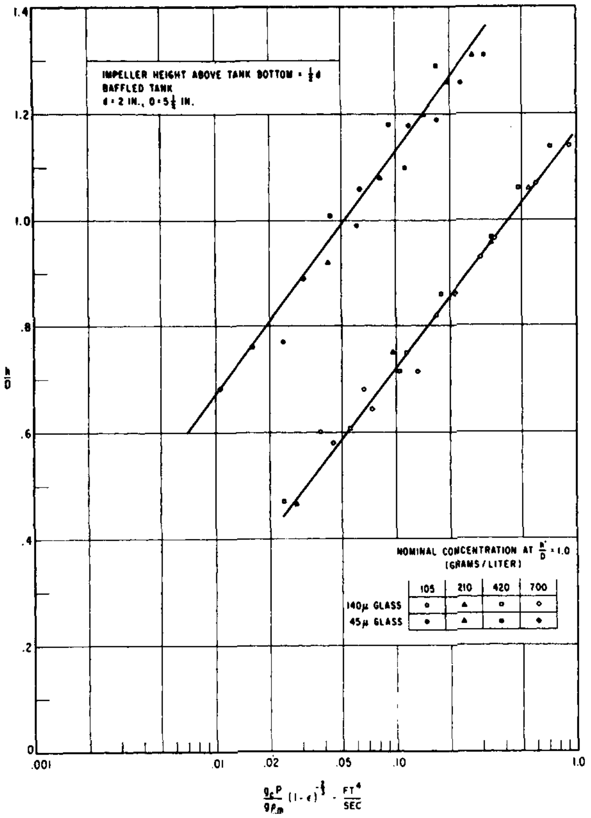


**Fig. 3. Effect of particle size on power requirements and height of the slurry interface.**

settling rate of the flocs to be of the order of 2 mm./sec. However with such a rapid settling rate the Andreasen pipette measurements are not accurate. The technique of using a substance such as glycerin to reduce the settling rate was not applicable in this case since another liquid would have changed the degree of flocculation. It will subsequently be shown that use of 2 mm./sec. for the free settling rate together with the assumption that the thoria flocs are half water allows the thoria data to be related to those obtained with glass beads. Additional support for this assumption is given by the work of Taylor and Biancheria (*1, 13*) in the correlation of room-temperature hindered settling rates with settled bed densities. They found that use of a 2 mm./sec. free settling rate and a 50% particle porosity in their equations resulted in a good correlation of the data for the thoria used in the present study.

## DISPERSION OF AQUEOUS SLURRIES IN THE TURBULENT REGION

The first problem studied was that of dispersing already suspended solids throughout the vessel volume. Zwietering's (*16*) and Hirsekorn and Miller's (*2*) observations that areas devoid of solids may exist adjacent to the air-liquid interface although all particles are suspended were confirmed, and a range of power inputs was found for which no particles remain on the tank bottom but a definite slurry-water interface exists. As the power input is increased, the interface rises until the slurry finally occupies the entire tank. The variation in concentration with distance below the interface was investigated by measuring the change in pressure with liquid depth at a number of positions. The measurements were made with manometer dip tubes and a sensitive differential pressure cell with a porous-tipped probe. The porous tip served to exclude slurry from the instrument lead line. The pressure cell was capable of detecting a pressure differential equivalent to a concentration change of 40 g./liter persisting for a depth of 1 in. Velocity-head effects were eliminated by making measurements during and immediately after mixing. Provided that no particles remained on the tank bottom, no departure from the linear relationship between pressure and depth was observed except in the immediate vicinity of the slurry-water interface. Once all solids are suspended, the slurry concentration, averaged across any plane parallel to the tank bottom, appears to be essentially uniform below the slurry interface. (It should be noted that the measurement technique used would not detect local concentration variations such as might occur adjacent to the impeller, impeller shaft, or vessel walls.) It was therefore concluded that the height of the slurry-water interface could be used as an index of the degree of slurry dispersion.

Most of the data reported in this paper were obtained with the total liquid height 1.4 times the tank diameter. However a number of runs were made in which the liquid depth was varied from 1 to 2 tank diam. The interface location and impeller power requirements were found to be independent of the liquid level above the interface.

The slurry-water interface was more clearly defined with increasing solids concentration and with a decreasing variation in particle size. In the turbulent region interface height determinations at solids concentrations below 100 g./liter were not reproducible and were therefore omitted from the present study.

The initial dispersion studies were carried out with glass-bead–water slurries by means of single impellers placed one-half an impeller diameter above the tank bottom. Reynolds numbers ($Nd^2\rho_l/\mu_l$) varied from about $2 \times 10^4$ to $2 \times 10^5$. These tests established that the power input increases exponentially with the height of the slurry interface above the impeller midplane. It was also found that at a given interface height the impeller power increases with increasing solids concentration. That this effect is not simply a matter of increasing mixture density is shown in Figure 2, where the interface height is plotted against $(g_c P)/(g\rho_m)$. It was found that the power varies directly with the volume fraction of solids to the 2/3 power $[(1-\epsilon)^{2/3}]$. The same variation was noted with the thoria slurries when $(1-\epsilon)$ was computed on the basis that the thoria flocs are half water. Application of the solids-fraction correction to a typical set of data is shown in Figure 3. This plot also shows the effect of particle size on power requirements. At a given interface level the data for the two particle sizes differ by a factor approximately equal to that of the ratio of the free-settling velocities of the particles.

The effects of both tank size and the ratio of impeller diameter to tank diameter were also evaluated. For geometrically similar systems compared at the same value of $h/D$ the power requirements were found to be directly proportional to the vessel volume. For values of $d/D$ between 0.176 and 0.426 it was found that the power per unit volume required to maintain the interface at a given value of $h/D$ varied inversely with $(d/D)^{1/2}$.

It was found that for a given system the interface height increases by 25 to 35% if the baffles are lifted from the bottom by a distance equal to ½ an impeller diameter. Thus a vessel baffled to the bottom is less efficient in terms of the power requirement to expand a bed. This aspect of mixing was not pursued further because of the limited time available for this study. All curves and equations presented in this paper are based on the case where the baffles extend to the tank bottom.

## ANALYSIS OF THE BEHAVIOR OF THE SLURRY INTERFACE

The experimental observations lead to the conclusion that the degree of mixing decreases with increasing distance from the impeller. It would seem reasonable to suppose that the region of the slurry-water interface is quiescent and therefore that laminar flow conditions prevail. In addition, at the slurry-water interface the suspending and settling forces must be in equilibrium. Making the above assumptions and noting that the particles just at the interface must be in a low concentration region, one can mathematically analyze the behavior of these particles. In so doing, the authors shall essentially follow the treatment developed by Langevin (*10*) for the analysis of Brownian motion.

For the interface region the partial differential equation describing the motion of a single particle in the vertical direction may be written as

$$\frac{m}{g_c}\left(\frac{\partial^2 x}{\partial t^2}\right) = -k\left(\frac{\partial x}{\partial t}\right) + X \quad (7)$$

Multiplying through by $x$ and manipulating to incorporate $x^2$ as the dependent variable leads to

$$\frac{m}{2\,g_c}\left[\frac{\partial^2(x^2)}{\partial t^2}\right] - \frac{m}{2\,g_c}\left(\frac{\partial x}{\partial t}\right)^2 = -\frac{k}{2}\left(\frac{\partial(x^2)}{\partial t}\right) + Xx \quad (8)$$

If one applies this to the behavior of an average particle, the last term becomes zero owing to the random variation of collision forces and particle displacements. Equation (8) then becomes

$$\frac{m}{2g_c}\left[\frac{\overline{\partial^2(x^2)}}{\partial t^2}\right] + \frac{k}{2}\left(\frac{\overline{\partial(x^2)}}{\partial t}\right) = \frac{m}{g_c}\left(\frac{\overline{\partial x}}{\partial t}\right)^2 \quad (9)$$

where the bar denotes averaged conditions. The last term of Equation (9) represents twice the vertical component of the average kinetic energy of a particle. By replacing $(m/g_c)\ [(\overline{\partial x})/(\partial t)]^2$ by $2\overline{E}_x$ and noting that at equilibrium conditions $t$ may be considered to be infinite, one may integrate Equation (9) to obtain

$$\overline{\Delta x^2} = \frac{2\overline{E}_x}{k/2}\,t \quad (10)$$

where $\overline{\Delta x}$ is the average vertical displacement of a particle in the time interval $t$. If one now evaluates the friction factor in terms of Stokes's free-settling velocity, he obtains

$$(\overline{\Delta x})^2 = \frac{4\,g_c\,\overline{E}_x\,v_s}{g\,V_p(\Delta\rho)}\,t \quad (11)$$

Division by $t^2$ and replacement of $\overline{E}_x/t$ by $p'/3$ (this assumes the $y$ and $z$ components of kinetic energy of the particle are equal to the $x$ component) yields

$$\frac{(\overline{\Delta x})^2}{t^2} = \frac{4\,g_c\,p'\,v_s}{3\,g\,V_p(\Delta\rho)} \quad (12)$$

The term $(\overline{\Delta x})^2/t^2$ is the square of the average velocity of a particle. Since the particle is in equilibrium at the slurry-liquid interface, the average velocity of a particle must be equal to the gravitational settling velocity. Therefore

$$p' = \frac{3\,g\,V_p(\Delta\rho)v_s}{4\,g_c} \quad (13)$$

It is now necessary to relate $p'$ to the power input of the impeller. It has been shown by a number of investigations (*12*) of the fully turbulent region, and borne out by the data obtained in this study, that for a given system and impeller speed the power input of the impeller is directly proportional to the mixture density. Hence the ratio of the power required to agitate the slurry mixture to that required to agitate the suspending medium is given by

$$\frac{P_l}{P} = \frac{\rho_l}{\rho_m} \quad (14)$$

It seems reasonable to conclude that the difference between $P$ and $P_l$ is the total power input to the solid particles:

$$P_p = P\,\frac{\Delta\rho}{\rho_m}\,(1-\epsilon) \quad (15)$$

If one designates the average power input to an individual particle as $p_{avg}$, then

$$p_{avg} = \frac{V_p\,P_p}{V(1-\epsilon)} = \frac{V_p\,P(\Delta\rho)}{V\,\rho_m} \quad (16)$$

For agitation in geometrically similar systems the ratio $p'/p_{avg}$ may be expected to be a function of $h/D$, $(1-\epsilon)$, and the mixing Reynolds number. However since in the fully turbulent region the Reynolds number does not affect the power number, it seems reasonable that for this restricted case $N_{Re}$ will not affect $p'/p_{avg}$. Therefore

$$p' = p_{avg}\cdot F\,(1-\epsilon)\cdot F'\,(h/D) = \left(\frac{V_p\,P(\Delta\rho)}{V\,\rho_m}\right) F''\,(1-\epsilon)\,F'\,(h/D) \quad (17)$$

Substitution of the value of $p'$ from Equation (13) and rearrangement gives

$$(h/D) = F''''\left(\frac{g_c P}{g\rho_m V v_s}\right)\cdot F'''(1-\epsilon) \quad (18)$$

## CORRELATION OF INTERFACE BEHAVIOR AT MIXING REYNOLDS NUMBER IN THE TURBULENT REGION

The previously cited experimental data show that the power requirements in geometrically similar systems vary directly with $(1-\epsilon)^{2/3}$ and exponentially with $h/D$. Inserting these functions explicitly in Equation (18) gives

$$(h/D) = K_1 \ln\left[\left(\frac{g_c\,P}{g\,\rho_m\,V\,v_s}\right)(1-\epsilon)^{-2/3}\right] + K_2 \quad (19)$$

If one extends the above equation to include the effect of varying impeller-diameter–to–tank-diameter ratios, one has

$$(h/D) = K'_1 \ln\left[\left(\frac{g_c P}{g \rho_m V v_s}\right)(1-\epsilon)^{-2/3}\cdot(d/D)^{1/2}\right] + K'_2 \quad (20)$$

Figure 4 shows the data for aqueous slurries agitated by a single impeller when correlated in this fashion. In this correlation the specific gravity of glass beads was taken as 2.25, and free-settling velocities in water of 18 and 1.85 mm./sec. were used for particle diameters of 140 and 45 $\mu$ respectively. As previously stated, the free-settling velocity of thoria particles was taken to be 2 mm./sec., and the porosity of thoria flocs was taken as ½ (specific gravity = 4.85). It can be seen that excellent agreement is obtained between the various systems tested. For $h/D$ values above 0.5 the equation of the average line through the data is

$$h/D = 0.23 \ln\left[\left(\frac{g_c P}{g \rho_m V v_s}\right)(1-\epsilon)^{-2/3}(d/D)^{1/2}\right] + 0.1 \quad (21)$$

To check the above correlation with another fluid, a few runs were conducted where the 140-$\mu$ glass beads at a nominal concentration of 105 g./liter were agitated in carbon tetrachloride. When correlated in accordance with Equation (22), the limited data obtained fall within the scatter observed with the water data.

The effect of the height of the impeller from the tank bottom, was investigated for several different systems. It was found that this effect depended on the ratio $a/d$. The results can be expressed in terms of an empirical correction factor, where

$$\frac{h}{D} = f\left\{0.23 \ln\left[\left(\frac{g_c P}{g \rho_m V v_s}\right)(1-\epsilon)^{-2/3}(d/D)^{1/2}\right] + 0.1\right\} \quad (22)$$

Values of $f$ as a function of $a/d$ are presented graphically in Figure 5. It will be seen that for $a/d$ values of ½ or more, $f$ remains at 1; at values of $a/d$ below ½, $f$ increases slightly.

It would be expected that the power to expand a suspended bed in a long tank would be materially reduced by the use of several impellers. The effect of multiple impellers on the slurry water interface level was therefore investigated. It was found that the correlation developed for single-impeller systems applies provided that the power is divided by the number of impellers and the interface height is measured above the midplane of the uppermost impeller. As shown in Figure 6, the data for one-, two-, and three-impeller systems, arranged as illustrated in Figure 1, are well correlated by

$$\frac{h}{D} = 0.23 \ln\left[\left(\frac{g_c P}{gn \rho_m V v_s}\right)(1-\epsilon)^{-2/3}(d/D)^{1/2}\right] + 0.1 \quad (23)$$

or

$$h/D = 0.23 \ln \Psi + 0.1 \quad (24)$$

Equation (24) can be used to determine, for the fully turbulent region the number of impellers required to produce a fully dispersed suspension with the minimum power input. The authors will consider the situation where there are $n$ equally spaced impellers; the lowest impeller is as close to the tank bottom as possible, and the distance of the liquid surface from the top impeller equals half the distance between impellers. It can be shown that the power is a minimum for

$$n = \frac{0.5 + 0.5\sqrt{1 + 0.92\, D/h'}}{-1 + \sqrt{1 + 0.92\, D/h'}} \quad (25)$$

For large values of $h'/D$ Equation (25) simplifies to

$$n = 2.18\ (h'/D) \quad (26)$$

When $h'/D$ is 5 or above, the value of $n$ computed by Equation (26) is within 10% of that computed by Equation (25).

The optimum spacing predicted by Equations (25) and (26) must be considered as only approximate. Use of these equations involves extrapolation of the power-input relationship somewhat beyond the range of the data obtained.

## DISPERSION STUDIES IN THE TRANSITION AND LAMINAR REGION

Although beyond the scope of the work originally contemplated, a number of runs were made outside the fully turbulent region. The work was restricted to studies of the 140/$\mu$ glass beads in the 5½-in. tank. Glycerol-water solutions having viscosities five and twenty-five times the viscosity of

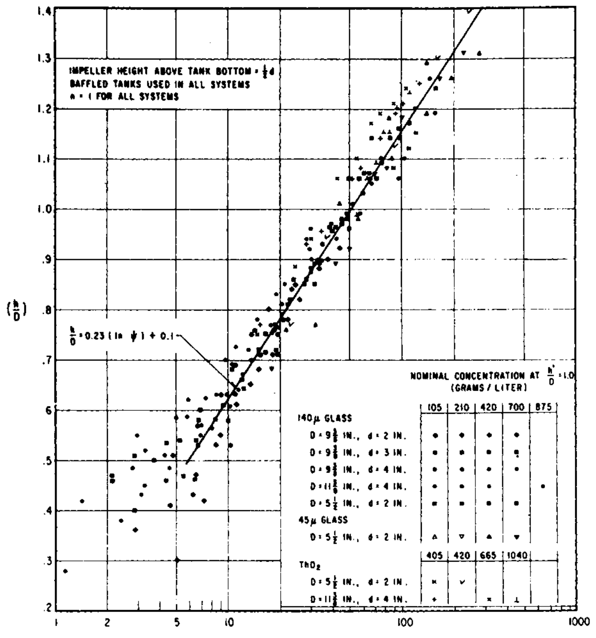


**Fig. 4. Generalized correlation of interface heights and power requirements for single impeller systems in the turbulent region.**

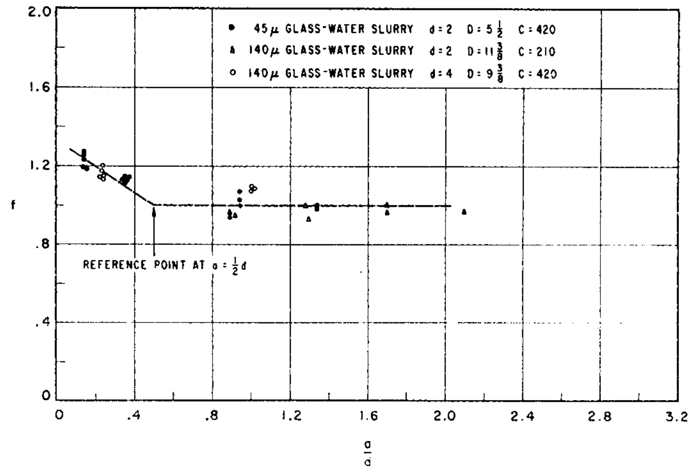


**Fig. 5. Interface height-correction factor as a function of impeller height above tank bottom.**

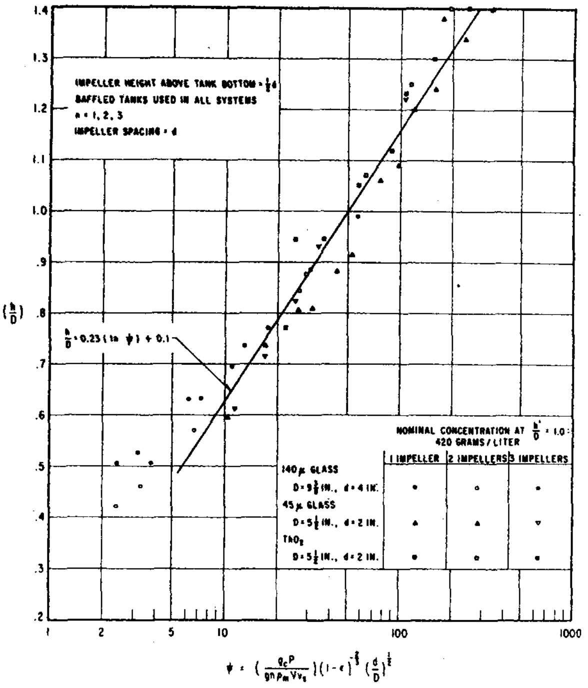

Fig. 6. Correlation of interface heights and power requirements for multiple impeller systems in the turbulent region.

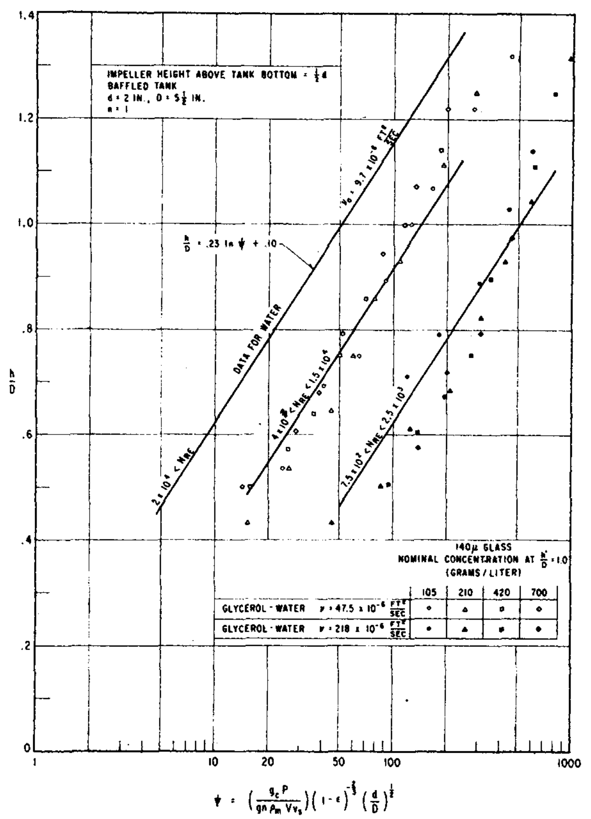

Fig. 7. Effect of Reynolds number on power requirements and interface heights.

room temperature water were used. Figure 7 compares the results with the data for the fully turbulent region. It is seen that the more viscous the suspending medium (lower Reynolds number), the more power is required. The supposition that the effect observed is a function of Reynolds number is also indicated by the fact that for a given fluid the increase in power is greatest at the lower interface heights and hence at the lowest agitator speeds and Reynolds numbers.

Figure 8 shows, as a function of Reynolds number, $Nd^2\rho_l/\mu_l$, the ratio of the actual power required to the power to maintain the same interface level computed from Equation (22). It would appear that this ratio is unity for values of $N_{Re}$ above $2.5 \times 10^4$. For values of $N_{Re}$ below $1.5 \times 10^4$ the data are fairly well correlated by

$$P/P_o = 4 \times 10^3 \, (N_{Re})^{-0.8} \quad (27)$$

The interface height data of Hirsekorn and Miller (2) for two-bladed paddles at $N_{Re}$ values below 25 are also shown on Figure 9. These data are also correlated by Equation (27). It will be noted that the ratio of $P/P_o$ begins to depart from unity at about $N_{Re} = 2 \times 10^4$. This is approximately the same Reynolds number at which the power number vs. $N_{Re}$ curves for many impellers cease being horizontal lines (12). The power data obtained in this study also follow this pattern.

## SUSPENSION-POWER REQUIREMENTS

The criterion used for full suspension is the same as that proposed by Hirsekorn and Miller (2) and Zwietering (16), namely that all particles are lifted from the vessel bottom. Since visual observation was required, it was necessary to rely solely on the glass-bead slurries for this portion of the investigation. All the thoria slurries studied were too milky to allow the point at which suspension occurred to be determined with any degree of accuracy.

Comparison of the data with the previously described correlations of Zwietering (16) and Kneule (9) showed that best agreement is obtained with Kneule's dimensionless group [Equation (3)]. If Equation (3) is rewritten with the notation of this paper used,

$$A' = \frac{V_t[g(\Delta\rho)]^{3/2}\delta^{1/2}}{P_s\, g_c\, \rho_l^{1/2}}\left(\frac{1-\epsilon_t}{\epsilon_t}\right)^{1/2} \quad (28)$$

Since in the turbulent region the relative velocity between the fluid and a particle is expressed by Newton's relation

$$u_s = 1.74\left[\frac{g\,\delta(\Delta\rho)}{\rho_l}\right]^{1/2} \quad (29)$$

Equation (28) may be rewritten as

$$A' = \frac{g\,V_t\,u_s(\Delta\rho)}{1.74\, g_c\, P_s}\left(\frac{1-\epsilon_t}{\epsilon_t}\right)^{1/2} \quad (30)$$

It will be noted that the dimensionless power group of Equation (30) is in the same form as the power group of Equations (18) through (23) but that $\Delta\rho$ replaces $\rho_m$ and $u_s$ replaces $v_s$. Use of a relative particle-fluid velocity calculated on the basis of Newton's law ($u_s$) rather than Stokes's law ($v_s$) seems reasonable since the particles on being removed from the tank bottom are in the turbulent region near the impeller.

The utility of Equation (31) in correlating the data from geometrically similar systems is shown in Figure 9. It will be noted that the data obtained with the glycerol-water solutions in the transition region are also correlated. It would appear that for mixing Reynolds numbers above about $10^3$ the relative velocity between the fluid and a particle at the tank bottom is sufficiently high so that Newton's law may be used. As expected, the data of Hirsekorn and Miller (2) for Reynolds numbers below 30 are not well correlated. The suspension powers required were approximately seven times those which would be indicated by the results of the present investigation.

Figure 10 shows the effect of varying the $D/d$ ratio as well as the height of the impeller above the tank bottom. For $D/d$ ratios of 2.35 to 2.75 the data are represented by

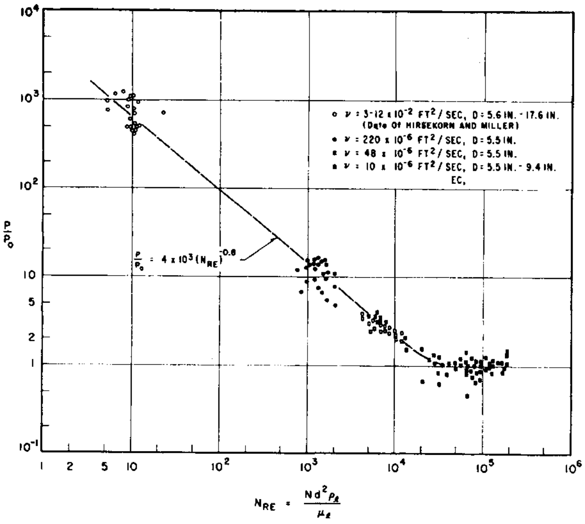


Fig. 8. Power requirement correction factor for the transition and laminar regions.

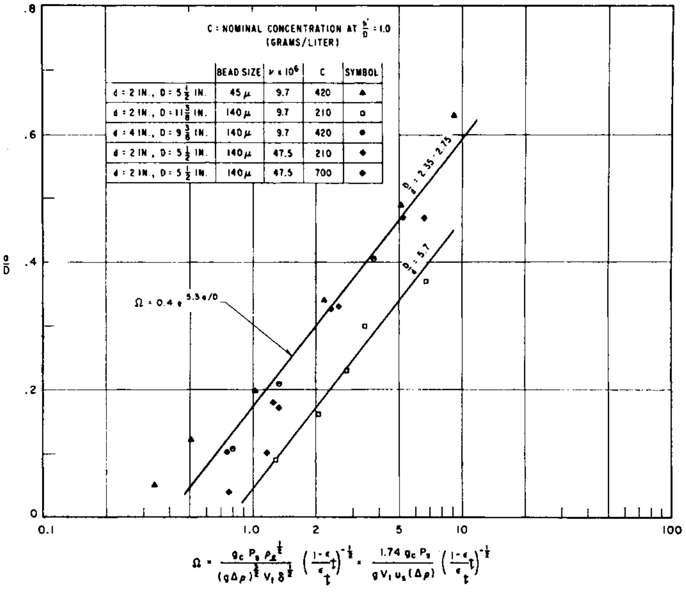


Fig. 10. Effect of system geometry on suspension power requirements.

$$\Omega = \frac{1.74\, g_c\, P_s}{g\, V_t\, u_s(\Delta\rho)} \left(\frac{1-\epsilon_t}{\epsilon_t}\right)^{-1/2} = 0.4\, e^{5.3\, a/D} \qquad (31)$$

For a $D/d$ ratio of 5.7, the data are represented by

$$\Omega = \frac{1.74\, g_c\, P_s}{g\, V_t\, u_s(\Delta\rho)} \left(\frac{1-\epsilon_t}{\epsilon_t}\right)^{-1/2} = 0.95\, e^{5.3\, a/D} \qquad (32)$$

Although a firm correlation cannot be advanced on the basis of the two $D/d$ ratios studied, the following tentative equation is suggested:

$$\frac{1.74\, g_c\, P_s}{g\, V_t\, u_s(\Delta\rho)} \left(\frac{1-\epsilon_t}{\epsilon_t}\right)^{-1/2} (d/D) = 0.16\, e^{5.3\, a/D} \qquad (33)$$

This is in agreement with the $(d/D)$ relationship observed by Hirsekorn and Miller [Equation (1)].

If Equation (32) is rewritten so that it is in the same form as Equation (21), one has

$$a/D = 0.19 \ln \left[ \frac{1.74\, g_c\, P_s}{g\, V_t\, u_s(\Delta\rho)} \left(\frac{1-\epsilon}{\epsilon}\right)_t^{1/2} (d/D) \right] + 0.36 \qquad (34)$$

The similarity in the effect of distance from the impeller shown in these equations seems reasonable. One would expect that the rate of power dissipation per unit volume would decrease with distance in the same fashion both above and below the impeller.

It is of interest to compare the results of Zwietering (*16*) for two-bladed paddles with the data of the present investigation. The dimensionless grouping used in this paper can be related to that used by Zwietering by

$$s\left(\frac{2.33\, N_p^{1/3} \rho_m^{1/3} \rho_s^{1/6} \nu^{0.1} \delta^{0.034} d^{0.81}}{\rho_l^{0.45} (g\Delta\rho)^{0.05} B^{0.036} D}\right) = \Omega^{1/3} \qquad (35)$$

As those quantities describing slurry properties either do not change greatly or have low exponents, the quantity within the parenthesis varies little over the range of slurry properties investigated by Zwietering. If comparison is based on a 10 wt. % slurry in water with particles having a specific gravity of 2.3 and a diameter of 300 $\mu$,

$$s\left(\frac{0.218\, N_p^{1/3} d^{0.81}}{D}\right) = \Omega^{1/3} \qquad (36)$$

A comparison of the values of $\Omega^{1/3}$ computed from the curves of Zwietering by means of Equation (36) and $\Omega^{1/3}$ computed by Equation (33) shows these to differ by a maximum of 30%. In addition while Zwietering had to use a separate set of curves for both paddle widths investigated, the present concept brings the curves for the two widths into close agreement.

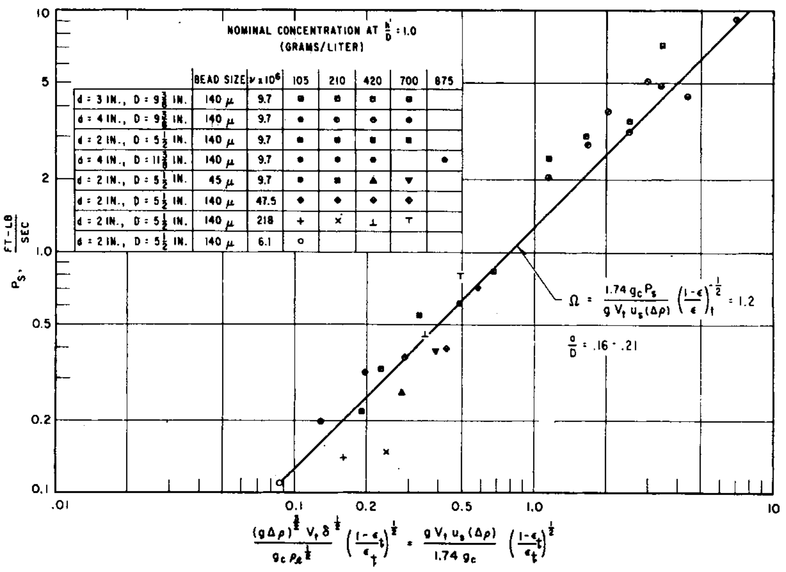


Fig. 9. Correlation of suspension power requirements for geometrically similar systems.

## SUMMARY

This study has confirmed earlier observations that the production of an essentially homogeneous slurry by rotating mixers involves both the initial suspension of the particles and the expansion of the particle bed to occupy the desired volume. At constant system geometry the initial suspension of the particles was found to be controlled by the dimensionless grouping $(g_cP)[(1-\epsilon)/\epsilon]^{-1/2}/[g\, V_t\, u_s(\Delta\rho)]$ first published by Kneule (*9*). For mixing

Reynolds numbers above $10^5$ Equation (33) can be used to predict suspension-power requirements when six-bladed paddles are used. The general agreement of Equation (33) with the results of Zwietering (*16*) indicates that suspension powers so calculated will also be approximately correct for paddles having fewer than six blades.

It was found that a range of power inputs existed where the slurry was fully suspended but where a definite slurry-fluid interface existed below the liquid level. The minimum power required to disperse a slurry in a given tank is thus determined by the conditions necessary just to maintain the slurry interface at the liquid level. The experimental results are consistent with the theoretical derivation which indicates that the interface level should be a function of the dimensionless group $(g_c P)/(g \rho_m V v_s)$. For a single six-bladed paddle located so that $a/D$ is at least 0.5 and operating at Reynolds numbers above $2.5 \times 10^4$, the relationship between interface height and power input is given by Equation (21). The same relationship is applicable to multiple impellers provided the interface height is measured above the top impeller and the power input is divided by the number of impellers. When $N_{Re}$ is below $2.5 \times 10^4$, the power required is greater than that computed by Equation (21) and the correction factor of Figure 8 is tentatively recommended.

Further work is needed in several areas. Additional studies with paddle type of impellers are needed to define more thoroughly behavior in the transition and laminar regions. Studies using solid-fluid systems with large density differences should be undertaken to establish definitely the density term appearing in the dimensionless power groups. These investigations should also be extended to other impeller types to determine whether similar relationships apply.

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## NOTATION

$a$ = height of lowest impeller above vessel bottom, ft.
$A$ = constant
$B$ = weight ratio of solids to liquid times 100 (referred to tank volume $V_t$),%
$c$ = solids concentration, g./liter of mixture
$d$ = impeller diameter, ft.
$D$ = vessel diameter, ft.
$E_x$ = vertical component of kinetic energy of single particle, ft.-lb.-force
$f$ = ratio of slurry interface height at given $a/d$ to height at $a/d = ½$, dimensionless
$F$ = functional relationship
$g$ = local gravitational acceleration, ft./sec.$^2$
$g_c$ = mass acceleration/force conversion factor, (lb.-mass)(ft.)/(lb.-force)(sec.$^2$)
$h$ = interface height above midplane of top impeller, ft.
$h'$ = interface height above vessel bottom, ft.
$k$ = friction factor for individual particle, (lb.-force)(sec.)/ft.
$K$ = constant
$m$ = mass of individual particle, lb.-mass
$n$ = number of impellers
$N$ = impeller speed (rev./time)
$N_p$ = power number = $P g_c/\rho_m N^3 d^5$, dimensionless
$N_{Re}$ = mixing Reynolds number = $Nd^2 \rho_l/\mu_l$, dimensionless
$N_i$ = number of particles in given size range
$p_{avg}$ = average power input to individual particle, ft.-lb./sec.
$p'$ = power input to a single particle in the region of the slurry water interface, ft.-lb.-force/sec.
$P$ = total mixing power input, ft. lb.-force/sec.
$P_l$ = power input to liquid, ft. lb.-force/sec.
$P_o$ = mixing power computed from Equation (22), ft. lb.-force/sec.
$P_p$ = power input to solid particles, ft. lb.-force/sec.
$P_s$ = minimum mixing power required to suspend solids, ft. lb.-force/sec.
$s$ = $\dfrac{N d^{0.85}}{\nu^{0.1}\delta^{0.2}(g\Delta\rho/\rho_l)^{0.45}B^{0.13}}$ dimensionless
$t$ = time, sec.
$u_s$ = relative vertical velocity between particle and fluid in turbulent region = $1.74 \left(\dfrac{g\delta(\Delta\rho)}{\rho_l}\right)^{1/2}$, ft./sec.
$v_s$ = free settling velocity calculated according to Stokes's law, ft./sec.
$V$ = system volume below interface, cu. ft.
$V_p$ = volume of individual particle, cu. ft.
$V_t$ = volume of vessel 1 diam. in height, cu. ft.
$X$ = force acting on particle owing to collisions, lb.-force
$x$ = vertical distance, ft.

### Greek Letters

$\delta$ = particle diameter, ft.
$\delta_{avg}$ = average diameter
$\epsilon$ = liquid fraction of system volume below interface, dimensionless
$\epsilon_t$ = liquid fraction based on vessel volume $V_t$, dimensionless
$\mu$ = size in $\mu$ (on figures)
$\mu_l$ = viscosity of suspending liquid, lb.-mass/sec. ft.
$\nu$ = kinematic viscosity, sq. ft./sec.
$\rho_m$ = density of slurry mixture below interface, lb.-mass/cu. ft.
$\rho_{mt}$ = density of slurry based on tank volume $V_t$, lb.-mass/cu. ft.
$\rho_l$ = density of suspending liquid, lb.-mass/cu. ft.
$\rho_s$ = density of solid, lb.-mass/cu. ft.
$\Delta\rho$ = $\rho_s - \rho_l$, lb.-mass cu. ft.
$\Psi$ = $\left(\dfrac{g_c P}{g n \rho_m V v_s}\right)(1-\epsilon)^{-2/3} (d/D)^{1/2}$, dimensionless
$\Omega$ = $\dfrac{1.74\, g_c P_s}{g V_t u_s(\Delta\rho)}\left(\dfrac{1-\epsilon_t}{\epsilon_t}\right)^{-1/2}$, dimensionless

## LITERATURE CITED

1. Garber, H. J., and G. R. Taylor, *WCAP*-1150, Westinghouse (1959).
2. Hirsekorn, F. S., and S. A. Miller, *Chem. Eng. Progr.*, **49**, 459 (1953).
3. Hixson, A. W., and S. J. Baum, *Ind. Eng. Chem.*, **33**, 478 (1941).
4. *Ibid.*, **34**, 120, 194 (1942).
5. Hixson, A. W., and J. H. Crowell, *ibid.*, **23**, 923, 1160 (1931).
6. Hixson, A. W., and G. A. Wilkens, *ibid.*, **25**, 1196 (1933).
7. Johnson, W. E., D. H. Fax, and S. C. Townsend, *Proc. Am. Power Conf.*, **19**, 640 (1957).
8. Johnson, W. E., and S. C. Townsend, *Nucleonics*, **17**, No. 2, 61 (1959).
9. Kneule, F., *Chem. Ing. Tech.*, **28**, 221 (1956).
10. Langevin, P., *Compt. Rend.*, **146**, 530 (1908).
11. Raghavendra, R. S., and B. K. Mukerji, *Trans. Indian Inst. Chem. Engrs.*, **7**, 63 (1954-55).
12. Rushton, J. H., E. W. Costich, and H. J. Everett, *Chem. Eng. Progr.*, **46**, 395, 467 (1950).
13. Taylor, G. R., and A.E. Biancheria, *WCAP*-1107, Westinghouse (1959).
14. White, A. M., and S. D. Summerford, *Ind. Eng. Chem.*, **25**, 1025 (1933).
15. ———, E. D. Bryant, and B. E. Lukens, *ibid.*, **24**, 1160 (1932).
16. Zwietering, T. N., *Chem. Eng. Sci.* **8**, 244 (1958).

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